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Hadronic Structure from Lattice QCD

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In this contribution I highlight recent lattice calculations of the nucleon structure functions and form factors with two flavors of dynamical quarks done by the QCDSF Collaboration.

Wisdom from Frank Wilczek

" ... the Higgs particle (or the doublet) is certainly *not* – depite much loose talk to the contrary – the Origin of Mass. (Still less is it the God Particle, whatever that means.) Most of the mass of ordinary matter is concentrated in protons and neutrons. It arises from an entirely different, and I think more profound and beautiful, source. Numerical simulation of QCD shows that if we built protons and neutrons in an imaginary world with no Higgs mechanism – purely out of quarks and gluons with zero mass – their masses would not be very different from what they actually are. Their mass mostly arises from pure energy, associated with the dynamics of confinement in QCD, according to relation $m = E/c^2$. *This profound account of the origin of mass is a crown jewel in our Theory of Matter*."

`hep-ph/0101187`

1 Introduction

The lattice formulation of QCD is at present the only known way of obtaining low energy properties of the theory in a direct way, i.e. without any model assumptions. Quantities within the grasp of lattice QCD involving light quarks include the hadron mass spectrum, quark masses, the Λ parameter, the chiral condensate, the nucleon sigma term, meson decay constants, the axial and tensor charge of the nucleon, form factors and moments of the polarized and unpolarized structure functions of the nucleon, pion and rho.

Our group, the QCDSF Collaboration[a], has been actively involved for the last few years in determining these quantities, all characterized by their non-perturbative nature. Rather

[a] S. Capitani, M. Göckeler, R. Horsley, W. Kürzinger, D. Pleiter, P. Rakow, G. Schierholz, A. Schiller, and collaborators.

than giving an exhaustive progress report of recent developments in the field I shall, due to lack of space, focus on two topics: nucleon form factors (including the axial charge) and moments of structure functions, and how the lattice method can lead to their determination. The structure functions bear the answer to the question posed by Wilczek, namely how quarks and gluons provide the binding (mass) and spin of the nucleon.

The lattice approach involves first euclideanizing the QCD action and then discretizing space-time (with lattice spacing a). The path integral then becomes a very high dimensional partition function, which is amenable to Monte Carlo methods of statistical physics. This allows correlation functions, which can be related to QCD matrix elements, to be determined.

Progress in the field is slow. First our 'box' must be large enough to fit our correlation functions into. Then, often a chiral extrapolation must be made from a quark mass region around the strange quark mass to the light up and down quarks. Furthermore, the continuum (i.e. $a \to 0$) limit must be taken. In addition to the above problems, to be able to compare with phenomenological or experimental results, matrix elements must be renormalized. In the statistical mechanics picture, we are approaching a second order phase transition, with all its attendant problems.

In the past, simply to save computer time, the fermion determinant in the action was discarded. This 'quenched' or 'valence' quark approximation is an uncontrolled approximation. Recently, however, simulations with two flavors of mass-degenerate sea quarks have begun appearing, allowing a first look at 'real' QCD and the effects of unquenching.

2 Generalities

Lepton–nucleon elastic scattering, $lN \to lN$, in which a photon is exchanged between the lepton (usually an electron) and the nucleon (usually a proton), has been studied for many years. Indeed, there has been a resurgence of interest in these processes as part of the Jefferson Laboratory physics program. The scattering matrix element can be decomposed into a known electromagnetic piece and an unknown QCD matrix element:

$$\langle \vec{p}\,', \vec{s}\,' |\, J_\mu(q)\, | \vec{p}, \vec{s} \rangle = \overline{u}(\vec{p}\,', \vec{s}\,') \left[\gamma_\mu F_1(Q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{2m_N} F_2(Q^2) \right] u(\vec{p}, \vec{s})\,, \qquad (1)$$

where $q = p' - p$ is the momentum transfer and $Q^2 = -q^2 > 0$. The values at $Q^2 = 0$ are $F_1^p(0) = 1$, $F_2^p(0) = \mu^p - 1$ for the proton and $F_1^n(0) = 0$, $F_2^n(0) = \mu^n$ for the neutron, where μ is the anomalous magnetic moment. Experimentally, it is more convenient to define the Sachs form factors

$$\begin{aligned} G_e(Q^2) &= F_1(Q^2) - \frac{Q^2}{(2m_N)^2} F_2(Q^2)\,, \\ G_m(Q^2) &= F_1(Q^2) + F_2(Q^2)\,. \end{aligned} \qquad (2)$$

Similarly, neutrino–nucleon scattering, for example $\nu_\mu n \to \mu^- p$ mediated by a W^+ exchange, leads to an unknown axial current hadronic matrix element between neutron and proton states, which, with the use of current algebra and isospin invariance, may be re-written

$$\langle \vec{p}\,', \vec{s}\,' |\, A_\mu^{u-d}(q)\, | \vec{p}, \vec{s} \rangle = \overline{u}(\vec{p}\,', \vec{s}\,') \left[\gamma_\mu \gamma_5 g_A(Q^2) + i\gamma_5 \frac{q_\mu}{2m_N} h_A(Q^2) \right] u(\vec{p}, \vec{s})\,, \qquad (3)$$

where $A_\mu^{u-d} = \overline{u}\gamma_\mu\gamma_5 u - \overline{d}\gamma_\mu\gamma_5 d$. From the β decay we know $g_A \equiv g_A(0) = 1.267(4)$.

At higher momentum transfer the nucleon is broken up by the photon (or $W^\pm$) probe, and we enter the regime of deep-inelastic scattering (DIS) experiments, $eN \to eX$ (or $\nu_\mu n \to \mu^- X$). The operator product expansion (OPE) leads to relations between moments of the structure functions and nucleon matrix elements of certain operators. For example[1],

$$\int_0^1 dx x^{n-2} F_2(x, Q^2) = \frac{1}{3} c_n(\mu^2/Q^2)\, v_n(\mu) + O(\frac{1}{Q^2})\,. \tag{4}$$

Here x is the Bjorken variable, c_n are the Wilson coefficients and $v_n \propto \langle N|\mathcal{O}_n|N\rangle$, where $\mathcal{O}_n = (i/2)^{n-1}\overline{q}\gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_n} q$ are operators bilinear in the quarks, each containing $n-1$ covariant derivatives. In parton model language

$$v_n = \langle x^{n-1} \rangle. \tag{5}$$

All matrix elements can be determined non-perturbatively using lattice QCD, while the Wilson coefficients, which contain the short-distance physics, can be computed in (continuum) perturbation theory. From the moments one can then reconstruct the structure functions.

3 Lattice Technicalities

In the last few years lattice QCD has improved in several respects, making it a quantitative tool of analysis.

Cut-off effects can be reduced (from $O(a)$) to $O(a^2)$ by adding irrelevant operators to the Wilson fermion action[2], S_F, and to the operators[3] whose matrix elements one wants to compute:

$$\begin{aligned} S_F &\to S_F - \frac{a}{4} c_{SW}\, g \sum_x \bar{\psi}(x)\sigma_{\mu\nu} F_{\mu\nu}(x)\psi(x)\,, \\ \mathcal{O} &\to (1 + c_0 am)\mathcal{O} + a \sum_{i\geq 1} c_i \mathcal{O}_i\,, \end{aligned} \tag{6}$$

with $c_{SW},\ c_0,\ c_1,\ \cdots$ to be determined with non-perturbative precision[4]. This greatly facilitates the extrapolation of the results to the continuum limit.

All calculations which I am going to present have been done with improved fermions (6) and for two flavors of dynamical quarks. The gauge field configurations have been genenerated in collaboration with UKQCD. Details of our present data sample can be found elsewhere[5].

The lattice operators (and matrix elements) are in general divergent and need to be renormalized, like the Wilson coefficients:

$$\mathcal{O}(\mu) = Z(\mu, a)\mathcal{O}(a). \tag{7}$$

There are several possibilities. The axial and vector renormalization constants may be determined by demanding that their (continuum) Ward identities are obeyed, which produces non-perturbative renormalization constants. Perturbation theory can be applied, but due to technical problems only one loop results are known. Even with this restriction, perturbation theory can be improved leading to 'tadpole improved' (TI) perturbation theory. However,

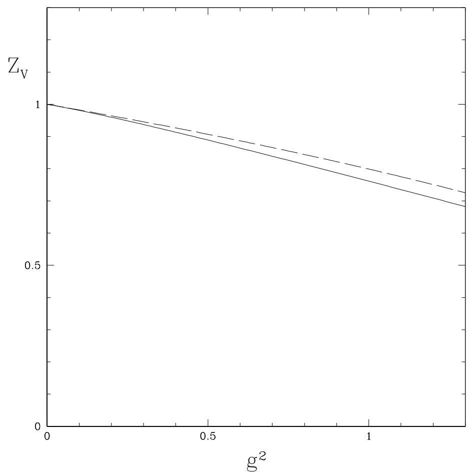

Figure 1. Z_V computed non-perturbatively (solid line) for two flavors of massless quarks and in TI perturbation theory (dashed line).

it still suffers from systematic errors. The QCDSF Collaboration has developed techniques allowing the renormalization constants to be determined non-perturbatively[6]. In Fig. 1 I compare the non-perturbative and TI perturbative renormalization constant for the vector current, which enters in the calculation of the form factors. We see that at our present couplings ($\beta \equiv 6/g^2 = 5.2 - 5.29$) TI perturbation theory would lead to a systematic error of $O(10\%)$.

At present, for the axial[7] and vector currents (see above) most renormalization constants and improvement coefficients are known non-perturbatively. For v_n we rely on TI perturbation theory[3].

4 Selected Results

We started only a year ago to look at the structure of the nucleon in the presence of dynamical quarks. Below I present some selected results.

The Axial Charge

The axial charge g_A of the nucleon is known very precisely experimentally. Hence it is a benchmark calculation of lattice QCD.

In Fig. 2 I show our results extrapolated to the continuum ($a = 0$) and chiral ($m_\pi = 0$) limits, using the fit formula

$$g_A = A + B(m_\pi r_0)^2 + C(a/r_0)^2 \,, \tag{8}$$

where $r_0 = 0.5\,\mathrm{fm} = 1/(395\,\mathrm{MeV})$ is the force parameter[8], and a correction term is introduced to account for possible $O(a^2)$ scaling violations, which are not removed by

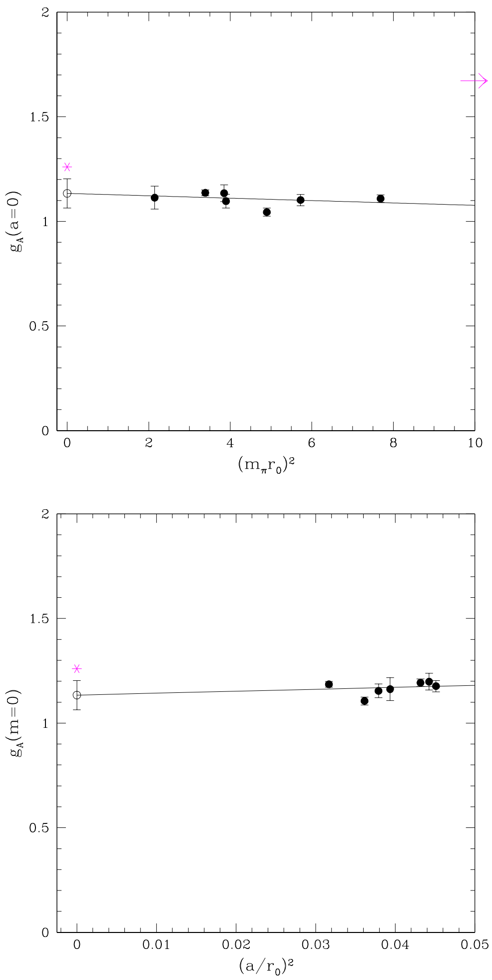

Figure 2. The chiral and continuum extrapolations of g_A for two flavors of dynamical quarks as a function of m_π and a, compared to the experimental value ($*$) and the heavy-quark limit ($\longrightarrow$).

our improvement program. The agreement with the experimental value is quite good. But it should be noted that we are still far away from the continuum and chiral limits. For comparison, in our latest quenched calculation the smallest lattice spacing was $(a/r_0)^2 \approx 0.01$. To go to similarly small lattice spacings in the dynamical case will require computers with a sustained speed of $O(10)$ Teraflop/s.

Form Factors

Nucleon form factors have been extensively studied, both experimentally and theoretically, for many years. They describe the overall distribution of electric, magnetic and axial charge

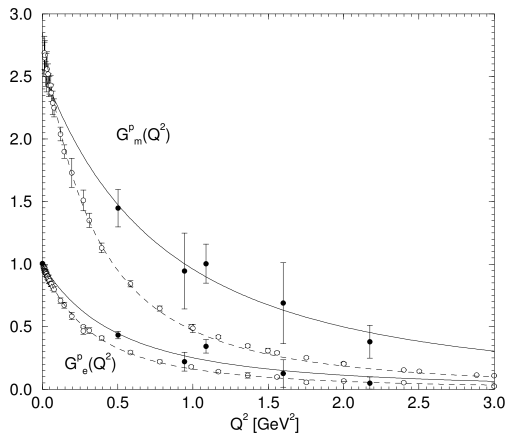

Figure 3. The electric and magnetic form factors of the proton, G_e and G_m (solid symbols), as a function of Q^2 together with the experimental numbers (open symbols).

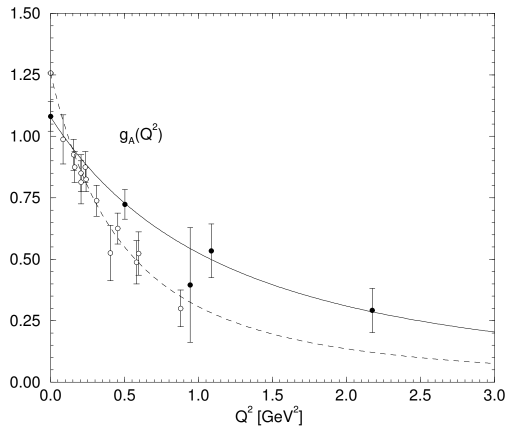

Figure 4. The axial form factor $g_A(Q^2)$ (solid symbols) as a function of Q^2 together with the experimental numbers (open symbols).

in the nucleon. An interesting aspect of lattice simulations is that one can change the value of the quark (pion) mass and study how the shape of the nucleon changes as one approaches the physical quark mass.

In Fig. 3 I show the electric and magnetic form factors of the proton for one particular coupling ($\beta = 5.25$) and a rather heavy pion mass, $m_\pi \approx 750\,\text{MeV}$. And in Fig. 4 I show the axial form factor. I compare the results with the experimental numbers. We see that the form factors of the lattice nucleon are shallower than their experimental counterparts, which indicates that the lattice nucleon is somewhat smaller than the physical one. Indeed, a fit of the radius gives $r_{\text{rms}} \approx 0.70\,\text{fm}$, which is to be compared with the phenomenolog-

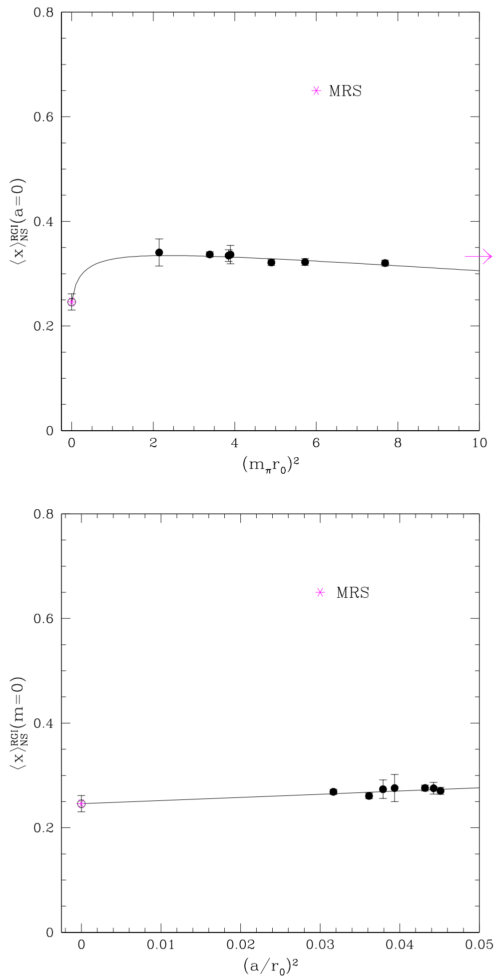

Figure 5. The chiral and continuum extrapolations of the fraction of the proton's momentum, $\langle x \rangle$, carried by the up quarks minus that of the down quarks, for two flavors of dynamical quarks as a function of m_π and a, compared to the experimental value ($*$) and the heavy-quark limit ($\longrightarrow$).

ical value $r_{\rm rms} = 0.83\,{\rm fm}$. This comes at no surprise. Quarks are tightly bound to each other, and they can leave the core of the nucleon practically only if they are bound to an anti-quark in form of a (light) pion. It happens then that pions are constantly emitted and reabsorbed, forming a pion cloud. And the smaller the pion mass is, the more extended is the pion cloud, and the larger is the charge radius of the nucleon.

At the moment our smallest pion mass is limited to $\gtrsim 500$ MeV. The reason is that the computational cost grows like $\sim 1/m_\pi^5$. The extrapolation of observables, which require an accurate view of the pion cloud, to the physical pion mass is difficult. It requires input from a low-energy effective theory of the nucleon, including pions and at least the $\Delta(1232)$

resonance. Furthermore, the pion mass must not be too large. For small Q^2 first analytic results on the pion mass dependence of the nucleon form factors are available[9]. We are currently attempting a chiral extrapolation incorporating these results.

Moments of the Nucleon Structure Function

A wealth of information on the structure of the nucleon is contained in the nucleon structure functions, unpolarized and polarized, and the quark and gluon distribution functions derived from them. These distribution functions give the probability of finding a quark or gluon with a certain momentum or spin in the nucleon. Lattice calculations of the lowest moment, $\langle x \rangle$, of the unpolarized quark distribution functions tells us what fraction of the nucleon's mass is carried by the quarks. Unbound quarks would only account for a few percent of the mass of the nucleon. The rest of the mass is due to the binding of the quarks and the gluons.

In Fig. 5 I show our results for $\langle x \rangle$. To compare the lattice results with the experimental number[10], one must extrapolate the data from the lowest calculated quark mass to the physical value, as I have discussed before. A simple, linear extrapolation overestimates the experimental number by $\approx 30\%$, suggesting that important physics is being omitted. Recently, it has been shown[11] that the nucleon's pion cloud gives rise to non-analytic terms in the quark mass, which may result in a large deviation from linearity as the quark mass tends to zero. A fit to the lattice data of the form

$$\langle x \rangle = A\Big(1 - \frac{3g_A^2 + 1}{(4\pi f_\pi)^2} m_\pi^2 \ln\big(\frac{m_\pi^2}{m_\pi^2 + \Lambda^2}\big)\Big) + B(m_\pi r_0)^2 + C(a/r_0)^2 \,, \qquad (9)$$

which preserves the correct (chiral) behavior and fits the experimental value, is also shown in Fig. 5.

These results have significant implications. It appears that calculations of the nucleon structure functions require the pion cloud to be adequately represented on the lattice. Even though one need not calculate at the physical quark (pion) mass, the pion must be light enough that the parameters of the chiral expansion are well determined by the lattice calculations. This is only the case for very light pions less than about 300 MeV, as can be infered from Fig. 5. Similar results are found for the higher moments.

5 Conclusions

To obtain quantitative results from lattice simulations, beyond hadron and quark masses, decay constants and perhaps Λ^5, is quite hard. But progress is steady. About five years ago, when we started to work on structure functions[1], our calculations were done with standard Wilson fermions on $16^3 32$ lattices in the quenched approximation and using perturbative renormalization constants. Today we are doing simulations in full QCD with two flavors of dynamical quarks on $24^3 48$ lattices, using a non-perturbatively improved fermion action, improved operators and non-perturbative renormalization constants.

The improvement program has paid off: discretization errors are found to be relatively small. But a continuum extrapolation is still indispensible, which requires to repeat the calculations at smaller lattice spacings.

To safely extrapolate to the chiral limit, we need to do simulations at smaller quark masses with $m_\pi \lesssim 300$ MeV, such as to include an accurate view of the pion cloud.

All this will require computers with a sustained speed of $O(10)$ Teraflop/s. But it should bring reliable calculations of hadronic structure within the capability of the next generation of computers such as the APE*next* machine.

Acknowledgements

The analysis of the dynamical gauge field configurations has been performed on the Cray T3E at NIC-Jülich, where we have used $\approx$ 100.000 CPU-hours (through September 2001), while the gauge field configurations themselves have been generated on the Hitachi SR-8000 at Leibniz Rechenzentrum in München and on the Cray T3E at the University of Edinburgh. We acknowledge fruitful discussions and collaboration with the UKQCD Collaboration.

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